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EFFICIENT RELIABILITY ANALYSIS OF MECHANICAL SYSTEMS UNDER DIFFERENT COV VALUES

Murat Mayda[1]

[1]Karamanoglu Mehmetbey University, Department of Mechanical Engineering, Karaman, Turkey

Abstract. Reliability analysis of mechanical systems under uncertainty requires high computational efforts. In such cases, carefully assigning the coefficient of variation (COV) to random design variables or, precisely and accurately determining the COV characteristic, can diminish the challenge. In this work, the effect of the COV value on an efficient reliability analysis is investigated by using a case study that is the reliability analysis of a piston ring. To obtain an efficient number of simulations for the sufficiently converged probability of failure value under different COV values, different number of MCS (Monte Carlo Simulation) realizations are tested computationally, and MCS realizations conducted since 500000 are observed to yield the sufficiently converged probability of failure value. Also, the results show that an increase in the COV value leads to an increase in probability of failure of the mechanical example. However, that increase in the probability of failure subjected to the increase in the COV value could not be the same in all of mechanical design problems. Therefore, careful consideration must be given to whether varying the COV values significantly affects the reliability analysis or not, and thus the COV values must be carefully determined or identified to obtain more accurate Pf values.

Keywords: *reliability analysis, coefficient of variation, efficiency, mechanical systems*.

Corresponding Author: *Murat Mayda, Karamanoglu Mehmetbey University, Yunusemre Yerleskesi 70100, Karaman, Turkey, +0903382262000, e-mail: mmayda@kmu.edu.tr*

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1. Introduction

Reliability analysis aims to ensure a probability of failure of a mechanical system does not exceed a specified limit. The reliability analysis of a mechanical system consisting of random variables or parameters is a challenging task since it requires computationally expensive simulations or complex procedures for approaching the accurate probability of failure (P_f) (Choi *et al.*, 2007; Mayda, 2017; Mayda & Choi, 2017). To that end, many research efforts have been carried out (Haeri & Fadaee, 2016; Lee & Kwak, 2006; Xu & Kong, 2018; Xue *et al.*, 2017). Most of these works have focused on different sampling algorithms and surrogate models. On the other hand, one of the means that are able to diminish the computational effort is to carefully assign the coefficient of variation (COV) to random design variables if it is unknown, or, if known, to precisely and accurately determine the COV characteristic because it have an important potential to influence the result of reliability analysis. In this work, the effect of the COV value on an efficient reliability analysis is investigated by following the main four steps: Establishing the design problem effectively (1), determining random and deterministic variables or parameters in the design problem (2), identifying the sufficient P_f and COV values (3) and finding the best efficient number of simulations by MCS (4). With the aim of obtaining an efficient number of simulations for the sufficiently converged P_f value under different COV values, different number of MCS realizations were tested computationally, and the corresponding convergence of P_f was observed in the scatter plots by using a case study that is the reliability analysis of a piston ring.

This paper is organized as follows: In Section 2, the reliability analysis process under different COV values, and the effect of COV on this process are theoretically described. A case study illustrating the reliability analysis of a piston ring under different COV values is carried out and the findings are discussed in Section 3. Eventually, the concluding remarks are included in Section 4.

2. Reliability analysis under different COV values

The *Coefficient of Variation* (COV), δ_X, which is known to be a measure of uncertainty or the relative amount of uncertainty in a variable, and is given by the following formula:

$$\delta_X = \frac{\sigma_X}{\mu_X} \tag{1}$$

where, σ_X is the standard deviation of a random variable *X*, and μ_X is the mean of *X*.

As for reliability analysis, this process concentrates on finding the probability of failures in a system when limit-state functions are exceeded. There are many reliability analysis methods, which are Monte Carlo Simulation (MCS), Importance Sampling, Latin Hypercube Sampling (LHS), First-order reliability methods, and Second-order reliability methods. In this work, MCS is chosen due to its simplicity and effective performance in both linear and non-linear design constraints. MCS aims to search an optimum or a true point by generating a large number of values from random variables. The probability of failure is calculated by:

$$P_f = \frac{N_f}{N_s} \tag{2}$$

where, N_{f} is the number of failures of a structural design when the given limit-state functions are exceeded. N_s is the total number of samples generated from the distribution specified.

In this work, the effect of COV on an efficient reliability analysis is investigated by following the main four steps:

1. *Establishing the design problem effectively*: a design problem, and its required constraints are carefully organized in such a way that the reliability analysis provides us with more useful and reliable solutions to the problem.
2. *Determining random and deterministic variables or parameters in the design problem*: determining whether random or not is a critical issue for the reliability analysis because the result of a reliability analysis with random variables might be significantly different than that with deterministic variables. For example, it would be better to consider the dimensions of a mechanical component as random variables since they are highly influenced by manufacturing tolerances or human factors.
3. *Identifying the sufficient P_f and COV values*: a P_f *ensuring* the sufficient reliability level for this design should be identified. At the meantime, COV

values within a large range must be specified to be able to see the accurate the effect of COV on the reliability analysis.

4. *Finding the best efficient number of simulations by MCS*: unnecessary large number of simulations refers to a very high computer cost; therefore, an efficient number of simulations must be examined for the sufficiently converged P_f value.

3. A case study: reliability analysis of a piston ring under different COV values

A piston ring has a mean diameter of 60 mm, a radial height *h* = 7 mm, a width *b* = 6 mm, and a distance or deflection δ by applying a force *F* as shown in Fig. 1. Herein, the deflection δ will be determined, based on the Castigliano's theorem. The applied force *F* and the Young's modulus *E* were assumed to be 200 N and131000 MPa, respectively.

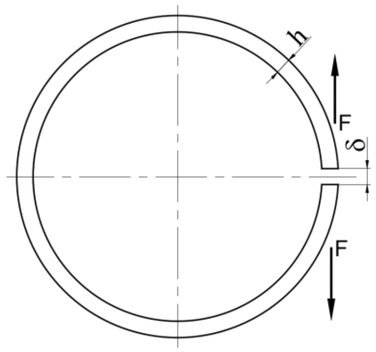

Figure 1. Piston ring design problem

$$\delta = \frac{3\pi F R^3}{EI} \tag{3}$$

where $I = bh^3$, R is the radius of the mean diameter (mm), F is the applied force (N), *h* and *b* are height and width of the ring, respectively. *E* is the Young's modulus.

From the design variables and parameters, *F*, *R* and *E* were assumed to be deterministic ones, and *b* and *h* were assumed to be random design variables. In this reliability analysis, the desired P_f was 0.01, and the COV range would vary from 1e-4 to 0.1. All of the required data for the reliability analysis is presented in Table 1.

Table 1. Statistical characteristics of the design variables and parameters used in the example

Design variables and parameters	Case of uncertainty	Type of distribution	COV
F	Deterministic	-	-
R	Deterministic	-	-
E	Deterministic	-	-
b	Random	Normal	From 1e-4 to 0.1
h	Random	Normal	From 1e-4 to 0.1

To explicitly indicate the variability of *b* and *h* variables with different COV values, normal distributions of them based on COV values of 0.05 and 0.1 using histogram are shown in Fig. 2a and 2b. The difference density and range of each variable under COV values is closely related to the degree to affect the reliability analysis.

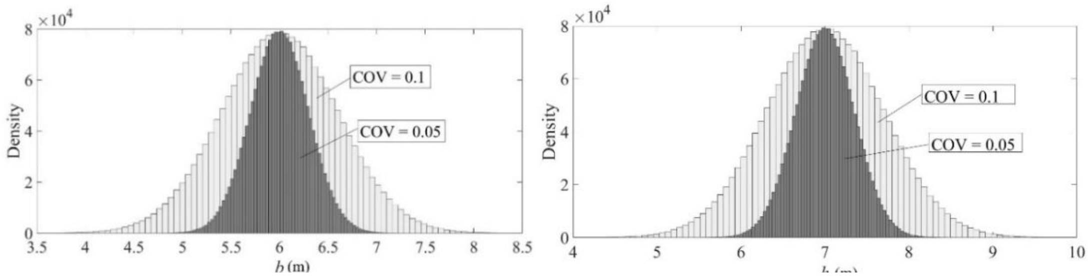

Figure 2. Normal distributions of *b* and *h* variables based on COV values of 0.05 and 0.1

With the aim of obtaining an efficient number of simulations for the sufficiently converged P_f value, different number of MCS realizations (10000, 50000, 100000, 500000, 1000000 and 2000000 realizations) were tested computationally, and the corresponding convergence of P_f was observed in the scatter plots (Fig. 3a, 3b, 3c, 3d, 3e and 3f). From these plots, it can be observed that the MCS realizations conducted since 500000 can be accepted to yield the sufficiently converged P_f value. In other words, 500000 MCS realizations are sufficient to obtain the true P_f value. Another point is that increasing the COV value in this design resulted in an increase in the P_f almost in all MCS realizations. However, that increase in the P_f subject to the increase in the COV value could not be the same in all of design problems. As a result, careful consideration must be given to whether varying the COV values significantly affects the reliability analysis or not. Accordingly, the COV values must be carefully determined or identified to obtain more accurate P_f values.

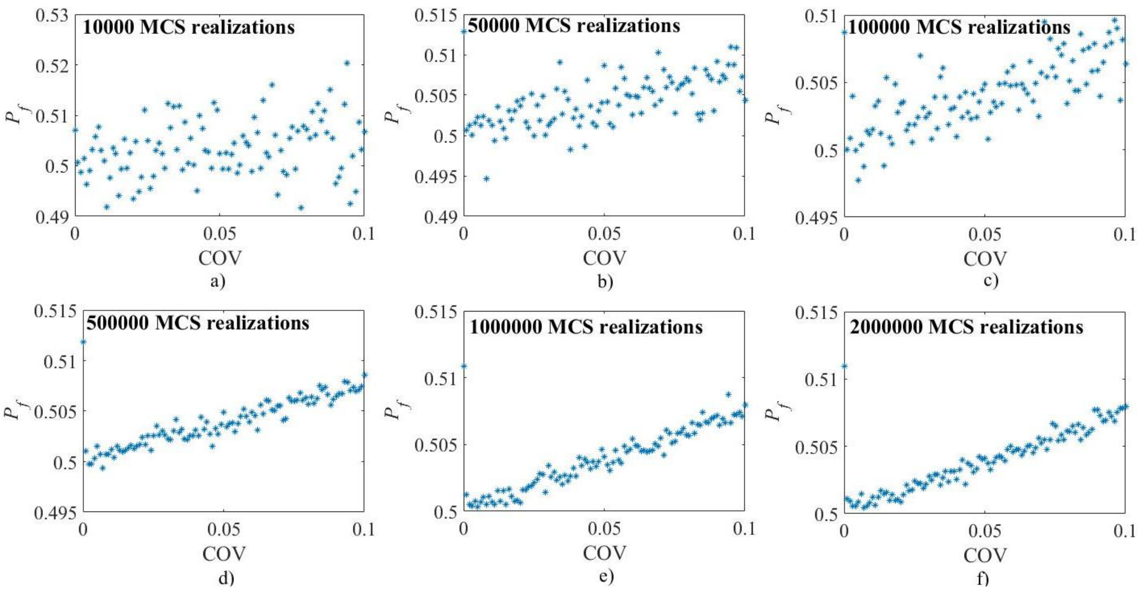

Figure 3. Probability of failure versus different coefficient of variation in different MCS realizations

4. Conclusion

In this work, the effect of COV on an efficient reliability analysis is investigated by following the main four steps by using a case study that is the reliability analysis of a piston ring. From the results of the computations, it can be observed that the MCS realizations conducted since 500000 can be accepted to yield the sufficiently converged P_f value. In other words, 500000 MCS realizations are sufficient to obtain the true P_f value. Another point is that increasing the COV value in this design resulted in an increase in the P_f almost in all MCS realizations. However, that increase in the P_f subject to the increase in the COV value could not be the same in all of mechanical design problems. As a result, careful consideration must be given to whether varying the COV values significantly affects the reliability analysis or not, and thus the COV values must be carefully determined or identified to obtain more accurate P_f values.

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